CHAPTER 5

ARITHMETIC PROGRESSIONS

Syllabus

- Motivation for studying Arithmetic Progression. Derivation of the n^{th} term and sum of the first n terms of A.P. and their application in solving daily life problems.

Trend Analysis

List of Concepts	2018		2019		2020	
	Delhi	Outside Delhi	Delhi	Outside Delhi	Delhi	Outside Delhi
Problems finding n^{th} term of the A.P.	1 Q (1 M)	1 Q (1 M)		1 Q (1 M) 1 Q (2 M) 1 Q (4 M)	2 Q (1 M) 2 Q (3 M)	2 Q (1 M) 2 Q (3 M)
Sum of n^{th} term of an AP	1 Q (2 M) 1 Q (4 M)	1 Q (2 M) 1 Q (3 M) 1 Q (4 M)		2 Q (2 M) 1 Q (4 M)	4 Q (3 M)	4 Q (3 M)
Word Problem on AP		1 Q (1 M)				

TOPIC - 1
To Find n^{th} Term of the Arithmetic Progression

Revision Notes

- An arithmetic progression is a sequence of numbers in which each term is obtained by adding or subtracting a fixed number d to the preceding term, except the first term.
- The difference between the two successive terms of an A.P. is called the common difference.
- Each number in the sequence of arithmetic progression is called a term of an A.P.
- The arithmetic progression having finite number of terms is called a finite arithmetic progression.
- The arithmetic progression having infinite number of terms is called an infinite arithmetic progression.

- A list of numbers $a_1, a_2, a_3, \ldots\ldots$ is an A.P., if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots$ give the same value *i.e.*, $a_{k+1} - a_k$ is same for all different values of k.
- The general form of an A.P. is $a, a + d, a + 2d, a + 3d, \ldots\ldots$
- If the A.P. $a, a + d, a + 2d, \ldots\ldots\ldots, l$ is reversed to $l, l - d, l - 2d, \ldots\ldots\ldots, a$, the common difference changes to negative of original sequence common difference.

Know the Formulae

- The general (n^{th}) term of an A.P. is expressed as:

$$a_n = a + (n-1)d \text{ .... from the starting.}$$

where, a is the first term and d is the common difference.
- The general (n^{th}) term of an A.P. $l, l - d, l - 2d, \ldots\ldots, a$ is given by:

$$a_n = l + (n-1)(-d) = l - (n-1)d \text{ .... from the end.}$$

where, l is the last term, d is the common difference and n is the number of terms.

Know the Terms

- A sequence is defined as an ordered list of numbers.
  The first, second and third terms of a sequence are denoted by t_1, t_2 and t_3 respectively.
- If the terms of sequence are connected with plus (+) or minus (–), the pattern is called a series.
  Example: 2 + 4 + 6 + 8 + ... is a series.
- The sequence of numbers 0, 1, 1, 2, 3, 5, 8, 13,...... was discovered by a famous Italian Mathematician *Leonasalo Fibonacci*, when he was dealing with the problem of rabbit population.
- If the terms of a sequence or a series are written under specific conditions, then the sequence or series is called a progression.
- If a constant is added or subtracted from each term of an A.P., the resulting sequence is also an A.P.
- If each term of an A.P. is multiplied or divided by a constant, the resulting sequence is also an A.P.
- If the n^{th} term is in linear form *i.e.*, $an + b = a_n$, the sequence is in A.P.
- If the terms are selected at a regular interval, the given sequence is in A.P.
- If three consecutive numbers a, b and c are in A.P., the sum two numbers is twice the middle number *i.e.*, $2b = a + c$.

How is it done on the GREENBOARD?

Q.1. Which term of the A.P. 6, 13, 20, 27, ... is 98 more than its 24^{th} term ?

Solution:

Step I: The given A.P. is 6, 13, 20, 27, ...

Here first term, $a = 6$

Common difference, $d = 13 - 6 = 7$

Step II: The 24^{th} term,

$$a_{24} = a + (24-1)d$$

or,

$$a_{24} = 6 + 23 \times 7$$
$$a_{24} = 6 + 161$$
$$a_{24} = 167$$

Step III: Now according to question,

$$a_{24} + 98 = a_n$$
$$167 + 98 = a + (n-1)d$$
$$265 = 6 + (n-1)7$$
$$259 = (n-1)7$$
$$\frac{259}{7} = n - 1$$
$$37 = n - 1$$

or $n = 38$

Hence, 38^{th} term is the required term.

Mnemonics

Concept: n^{th} Term of Arithmetic Progressio $n = a + (n - 1)d$.

Nokia Offers Additional Programmers in English To Attract Positive New One Buyer Daily

Interpretation:

Nokia's 'N' is ***n*^th^** term.
Offer's 'O' is **of**
Additional's 'A' is Arithmetic Programmer's 'P' is Progression
In's 'I' is **is**.
English's 'E' is **Equal**
To's 'T' is **To**
Attract's 'A' is ***a***
Positive's 'P' is **+**
New's 'N' is ***n***
One buyer is **– 1**
Daily's 'D' is *d*

Very Short Answer Type Questions — 1 mark each

AI Q. 1. Which term of the following A.P. 27, 24, 21, ... is zero ? A [CBSE SQP, 2020-21]

Sol. We know that
$a_n = a + (n-1)d$
$l = 0$
$0 = 27 + (n-1)(-3)$ ½
$30 = 3n$
$n = 10$ ½
10^{th} term of the given A.P. is zero.
[CBSE Marking Scheme, 2020-21]

Detailed Solution:

Given A.P. = 27, 24, 21, ...
Here, $a = 27$ and $d = 24 - 27 = -3$
and, $l = 0 = a_n$
$\therefore \quad a_n = a + (n-1)d$ ½
$\Rightarrow \quad 0 = 27 + (n-1)(-3)$
$\Rightarrow \quad -3n + 3 = -27$
$\Rightarrow \quad -3n = -27 - 3 = -30$
$\Rightarrow \quad n = 10.$ ½

AI Q. 2. In an Arithmetic Progression, if $d = -4$, $n = 7$, $a_n = 4$, then find a. A [CBSE SQP, 2020-21]

Sol. We know that
$a_n = a + (n-1)d$
$4 = a + 6 \times (-4)$ ½
$a = 28$ ½
[CBSE Marking Scheme, 2020-21]

Detailed Solution:

We have, $d = -4$, $n = 7$, and $a_n = 4$
$\therefore \quad a_n = a + (n-1)d$ ½
$\Rightarrow \quad 4 = a + (7-1)(-4)$
$\Rightarrow \quad 4 = a + 6(-4) = a - 24$
$\Rightarrow \quad a = 4 + 24$
$\Rightarrow \quad a = 28.$ ½

AI Q. 3. Find the value of x for which $2x$, $(x + 10)$ and $(3x + 2)$ are the three consecutive terms of an A.P. R [CBSE Delhi, Set-I, 2020]

Sol. $\because$ $2x$, $(x + 10)$ and $(3x + 2)$ are in A.P.
$\Rightarrow \quad (x + 10) - 2x = (3x + 2) - (x + 10)$ ½
$\Rightarrow \quad -x + 10 = 2x - 8$
$\Rightarrow \quad -x - 2x = -8 - 10$
$\Rightarrow \quad -3x = -18$
Hence, $\quad x = 6.$ ½

Q. 4. If the first term of an A.P. is p and the common difference is q, then find its 10^{th} term. R [CBSE Delhi, Set-I, 2020]

Sol. We have, first term $(a) = p$,
Common difference $(d) = q$
and $\quad n = 10$
Then, $\quad a_n = a + (n-1)d$ ½
$\Rightarrow \quad a_{10} = p + (10-1)q$
$\Rightarrow \quad a_{10} = p + 9q.$ ½

Q. 5. Find the common difference of the A.P. $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}$, ... R [CBSE OD Set-I, 2020]

Sol. Given A.P. = $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p} \ldots$

Here, let $\quad a_1 = \frac{1}{p}$ and $a_2 = \frac{1-p}{p}$

$\therefore$ Common difference $= a_2 - a_1 = \frac{1-p}{p} - \frac{1}{p}$

$= \frac{1-p-1}{p}$

$= \frac{-p}{p}$

$= -1.$ 1

AI **Q. 6. Find the n^{th} term of the A.P. $a, 3a, 5a, ...$.**

A [CBSE SQP, 2020-21]

Sol. Given A.P. $= a, 3a, 5a, ...$

Here first term, $a = a$ and $d = 3a - a = 2a$ ½

$\therefore$ n^{th} term $= a + (n-1)d$

$= a + (n-1)2a$

$= a + 2na - 2a$

$= 2na - a$

$= (2n-1)a.$ ½

Q. 7. How many two digits numbers are divisible by 3 ?

U [CBSE Delhi Set-1, 2019]

Sol. Numbers are 12, 15, 18, ..., 99 ½

$\therefore$ $99 = 12 + (n-1) \times 3$

$\Rightarrow$ $n = 30$ ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Numbers divisible by 3 are 3, 6, 9, 12, 15, -------, 96, 99

Lowest two digit number divisible by 3. is 12. and highest two digit number divisible by 3 is 99.

Hence, the sequence start with 12 ends with 99 and common difference is 3.

So, the A.P. will be 12, 15, 18, ----, 96, 99

Here, $a = 12$, $d = 3$, $l = 99$

$\therefore$ $l = a + (n-1)d$

$\therefore$ $99 = 12 + (n-1)3$

$\Rightarrow$ $99 - 12 = 3(n-1)$

$\Rightarrow$ $n - 1 = \dfrac{87}{3}$

$\Rightarrow$ $n - 1 = 29$

$\Rightarrow$ $n = 30$

Therefore, there are 30, two digit numbers divisible by 3.

Q. 8. In an A.P., if the common difference $(d) = -4$, and the seventh term (a_7) is 4, then find the first term.

U [Delhi/OD, 2018]

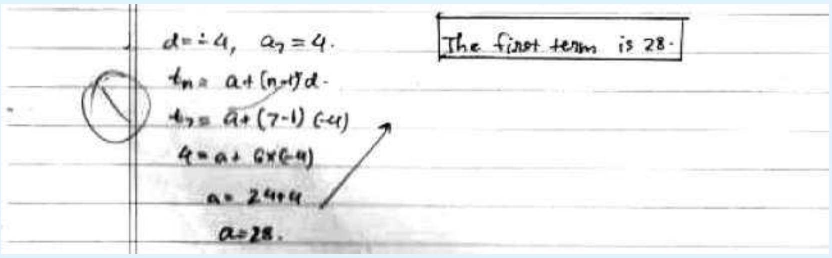
Sol. Since, $a + 6(-4) = 4$

$\Rightarrow$ $a = 28$ 1

[CBSE Marking Scheme, 2018]

Detailed Solution:

Topper Answer, 2018



$d = -4$, $a_7 = 4$.
$t_n = a + (n-1)d$.
$t_7 = a + (7-1)(-4)$
$4 = a + 6 \times (-4)$
$a = 24 + 4$
$a = 28$.

The first term is 28.

Q. 9. Which term of the A.P. 8, 14, 20, 26, ... will be 72 more than its 41^{st} term.

A [CBSE Outside Delhi Set-II 2017]
[CBSE Board Comptt. Set-III, 2017]

Sol. Given $a = 8$ and $d = 6$.

Let n^{th} term be 72 more than its 41^{th} term.

$\therefore$ $t_n - t_{41} = 72$

$8 + (n-1)6 - (8 + 40 \times 6) = 72$

$8 + (n-1)6 = 320$

$(n-1)6 = 312$

$n - 1 = 52$

$n = 53$ 1

AI **Q. 10. Write the n^{th} term of the A.P. $\dfrac{1}{m}, \dfrac{1+m}{m}, \dfrac{1+2m}{m}, ...$**

A [CBSE Delhi Comptt. Set-I, II, III, 2017]

Sol. We have, $a = \dfrac{1}{m}$

$d = \dfrac{1+m}{m} - \dfrac{1}{m} = 1$

$\therefore$ $a_n = \dfrac{1}{m} + (n-1)1$

Hence, $a_n = \dfrac{1}{m} + n - 1$

$= \dfrac{1+(n-1)m}{m}$ 1

Q. 11. If the n^{th} term of the A.P. $-1, 4, 9, 14, ...$ is 129. Find the value of n.

A [CBSE Delhi Comptt. Set I, II, III, 2017]

Sol. Given, $a = -1$ and $d = 4 - (-1) = 5$

$a_n = -1 + (n-1) \times 5 = 129$ ½

or, $(n-1)5 = 130$

$(n-1) = 26$

$n = 27$

Hence, 27^{th} term $= 129$. ½

[CBSE Marking Scheme, 2017]

Q. 12. **What is the common difference of an A.P. in which $a_{21} - a_7 = 84$?** A [CBSE Outside Delhi Set-I, II, III, 2017]

Topper Answer, 2017

Sol.

Let a be 1st term and d be the common difference.

$a_{21} - a_7 = 84$

$a + (21-1)d - [a + (7-1)d] = 84$

$a + 20d - a - 6d = 84$

$14d = 84$

$d = 6$

$\therefore$ common difference is 6.

Q. 13. **For what value of k will $k + 9$, $2k - 1$ and $2k + 7$ are the consecutive terms of an A.P. ?**

C + A [OD Set II, 2016]

Topper Answer, 2016

Sol.

We have-

Three consecutive terms of AP = $k+9$, $2k-1$, $2k+7$

Then,

$(k+9)+(2k+7) = 2(2k-1)$ $\{(a+c=2b)\}$

$\Rightarrow k+9+2k+7 = 4k-2$

$3k+16 = 4k-2$

$16+2 = 4k-3k$

$18 = k$

1

Q. 14. Find the tenth term of the sequence: $\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots$.

U [SQP, 2016] [Foreign Set-I, II, III, 2015]

Sol. Given sequence is an A.P.

$$\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots = \sqrt{2}, 2\sqrt{2}, 3\sqrt{2} \ldots$$

Hence, $a = \sqrt{2}$, $d = \sqrt{2}$ and $n = 10$

$\because$ $a_n = a + (n-1)d$

or, $a_{10} = \sqrt{2} + (10-1)\sqrt{2}$

$= \sqrt{2} + 9\sqrt{2}$

$= 10\sqrt{2}$

Hence, $a_{10} = \sqrt{200}$. 1

Q. 15. Is series $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$ an A.P. ? Give reason.

U [CBSE, Term-2, 2015]

Sol. Common difference,

$d_1 = \sqrt{6} - \sqrt{3}$

$= \sqrt{3}(\sqrt{2} - 1)$

Again, $d_2 = \sqrt{9} - \sqrt{6}$

$= 3 - \sqrt{6}$

$d_3 = \sqrt{12} - \sqrt{9}$

$= 2\sqrt{3} - 3$

As common differences are not equal.
Hence, the given series is not an A.P.

[CBSE Marking Scheme, 2015] 1

Short Answer Type Questions-I

2 marks each

AI **Q. 1. Find the number of natural numbers between 102 and 998 which are divisible by 2 and 5 both.**

A [CBSE SQP, 2020]

Sol. 110, 120, 130, ..., 990

$a_n = 990 \Rightarrow 110 + (n-1) \times 10 = 990$ 1

$\therefore$ $n = 89$ 1

[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

The number which ends with 0 is divisible by 2 and 5 both.

$\therefore$ Such numbers between 102 and 998 are:

110, 120, 130, ..., 990.

Last term, $a_n = 990$

$a + (n + 1)d = 990$ **1**

$110 + (n - 1) \times 10 = 990$

$110 + 10n - 10 = 990$

$10n + 100 = 990$

$10n = 990 - 100$

$10n = 890$

$n = \frac{890}{10} = 89.$ **1**

AI **Q. 2. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in A.P. .** A [CBSE Delhi Set-I, 2020]

Sol. Given: $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$

Common difference,

$d_1 = (a^2 + b^2) - (a - b)^2$
$= a^2 + b^2 - (a^2 + b^2 - 2ab)$
$= a^2 + b^2 - a^2 - b^2 + 2ab$
$= 2ab$ **½**

and $d_2 = (a + b)^2 - (a^2 + b^2)$
$= a^2 + b^2 + 2ab - a^2 - b^2$
$= 2ab$ **½**

Since, $d_1 = d_2$

Hence, $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are is an A.P. **1**

Hence Proved.

Q. 3. Which term of the A.P. 3, 15, 27, 39,... will be 120 more than its 21st term ? A [CBSE Delhi Set-I, 2019]

Sol. $a_n = a_{21} + 120$
$= (3 + 20 \times 12) + 120$
$= 363$ **1**

$\therefore$ $363 = 3 + (n - 1) \times 12$

$\Rightarrow$ $n = 31$ **1**

or 31st term is 120 more than a_{21}.

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given A.P. is: 3, 15, 27, 39

Here, first term, $a = 3$ and common difference, $d = 12$

Now, 21st term of A.P. is

$t_{21} = a + (21 - 1)d$ $[t_n = a + (n - 1)d]$

$\therefore$ $t_{21} = 3 + 20 \times 12 = 243$

Therefore, 21st term is 243

We need to calculate term which is 120 more than 21st term

i.e., it should be 243 +120 = 363 **1**

Therefore, $t_n = 363$

$\therefore$ $t_n = a + (n - 1)d$

$\Rightarrow$ $363 = 3 + (n - 1)12$

$\Rightarrow$ $360 = 12(n - 1)$

$\Rightarrow$ $n - 1 = 30$

$\Rightarrow$ $n = 31$

So, 31st term is 120 more than 21st term. **1**

Q. 4. Find the 20th term from the last term of the A.P.: 3, 8, 13, ... 253. A [CBSE SQP, 2018]

Sol. 20th term from the end $= l - (n - 1)d$ **½**

$= 253 - 19 \times 5$ **1**

$= 158$ **½**

[CBSE Marking Scheme, 2018]

Detailed Solution:

Given A.P.: 3, 8, 13, ... 253

Here, first term $(a) = 3$, common difference $(d) = 8 - 3 = 5$ and last term $(l) = 253$ **1**

Then, 20th term from the end of the A.P.

$= l - (n - 1)d$
$= 253 + (20 - 1)5$
$= 253 - 95$
$= 158.$ **1**

Q. 5. If 7 times the 7th term of an A.P. is equal to 11 times its 11th term, then find its 18th term. A [CBSE SQP-2018] [Foreign Set-2017] [CBSE Board Term-II, 2016]

Sol. $7a_7 = 11a_{11}$

$\Rightarrow$ $7(a + 6d) = 11(a + 10d)$ **1**

$\Rightarrow$ $a + 17d = 0$

$\therefore$ $a_{18} = 0$ **1**

[CBSE Marking Scheme, 2018]

Detailed Solution:

Given, $7a_7 = 11a_{11}$

$\because$ $a_n = a + (n - 1)d$

Then, $7[a + (7 - 1)d] = 11[a + (11 - 1)d]$

$\Rightarrow$ $7(a + 6d) = 11(a + 10d)$

$\Rightarrow$ $7a + 42d = 11a + 110d$

$\Rightarrow$ $11a - 7a = 42d - 110d$

$\Rightarrow$ $4a = -68d$

$\Rightarrow$ $a = -17d$ **1**

$\Rightarrow$ $a + 17d = 0$

i.e., $a + (18 - 1)d = 0$

Hence, $a_{18} = 0.$ **1**

Q. 6. Find how many integers between 200 and 500 are divisible by 8. A [Board Delhi comptt. Set-I, II, III, 2017]

Sol. Integers divisible by 8 are 208, 216, 224, ..., 496. **1**

Which is an A.P.

Given: $a = 208$, $d = 8$ and $l = 496$

Let the numbers of terms in A.P. be n.

$\because$ $a_n = a + (n - 1)d = l$

$\therefore$ $208 + (n - 1)d = 496$

$(n - 1)8 = 496 - 208$ **½**

$n - 1 = \frac{288}{8}$

$= 36$

$n = 36 + 1 = 37$ **½**

Hence, no. of required integers divisible by 8 = 37.

Q. 7. The fifth term of an A.P. is 26 and its 10th term is 51. Find the A.P.

A [OD Comptt. Set-II, 2017]

Sol. Here, $a_5 = a + 4d = 26$ ...(i) ½

and $a_{10} = a + 9d = 51$ ...(ii) ½

Solving Eqns. (i) and (ii), we get

or, $5d = 25$

$d = 5$ ½

and $a = 6$

Hence, the A.P. is 6, 11, 17 .... ½

[CBSE Marking Scheme, 2017]

Q. 8. How many two digit numbers are divisible by 7 ?

A [CBSE SQP, 2016]

Sol. Two digit numbers which are divisible by 7 are: 14, 21, 28, ..., 98. ½

It forms an A.P.

Here, $a = 14, d = 7$ and $a_n = 98$ ½

Since, $a_n = a + (n-1)d$

$98 = 14 + (n-1)7$ ½

$98 - 14 = 7n - 7$

$84 + 7 = 7n$

or, $7n = 91$

or, $n = 13$ ½

[CBSE Marking Scheme, 2016]

AI **Q. 9. In a certain A.P. 32nd term is twice the 12th term. Prove that 70th term is twice the 31st term.**

A [Board Term-2, 2015]

Sol. Let the 1st term be a and common difference be d.

According to the question, $a_{32} = 2a_{12}$

$\therefore$ $a + 31d = 2(a + 11d)$

$a + 31d = 2a + 22d$

$a = 9d$ 1

Again, $a_{70} = a + 69d$

$= 9d + 69d = 78d$

$\because$ $a_{31} = a + 30d$

$= 9d + 30d = 39d$

Hence, $a_{70} = 2a_{31}$ **Hence Proved.** 1

[CBSE Marking Scheme, 2015]

AI **Q. 10. The 8th term of an A.P. is zero. Prove that its 38th term is triple of its 18th term.**

A [CBSE Board Term-2, 2015]

Sol. Given, $a_8 = 0$ or, $a + 7d = 0$ or, $a = -7d$ ½

or, $a_{38} = a + 37d$

or, $a_{38} = -7d + 37d = 30d$ ½

And, $a_{18} = a + 17d$

$= -7d + 17d = 10d$ ½

or, $a_{38} = 30d = 3 \times 10d = 3 \times a_{18}$

$\therefore$ $a_{38} = 3a_{18}$. **Hence Proved.** ½

[CBSE Marking Scheme, 2015]

Q. 11. The fifth term of an A.P. is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference.

A [Foreign Set II, 2015]

[CBSE Board Term-II, 2015]

Sol. Let the first term be a and common difference be d.

Then, $a + 4d = 20$ ...(i) ½

and $a + 6d + a + 10d = 64$

$a + 8d = 32$ ...(ii) 1

Solving equations (i) and (ii), we get

$d = 3$

Hence, common difference, $d = 3$ ½

[CBSE Marking Scheme, 2015]

Q. 12. Find the middle term of the A.P. 213, 205, 197, ... 37.

A [CBSE Delhi Board Term, 2015]

Sol. Here, $a = 213$, $d = 205 - 213 = -8$ and $l = 37$

Let the number of terms be n.

$\because$ $l = a + (n-1)d$

$\therefore$ $37 = 213 + (n-1)(-8)$

or, $37 - 213 = -8(n-1)$

or, $n - 1 = \dfrac{-176}{-8} = 22$ ½

or, $n = 22 + 1 = 23$ ½

The middle term will be $= \dfrac{23+1}{2} = 12^{th}$ ½

$\therefore$ $a_{12} = a + (n-1)d$

$= 213 + (12-1)(-8)$

$= 213 - 88$

$= 125$

Thus, the middle term will be 125. ½

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-II — 3 marks each

AI **Q. 1. Which term of the A.P. $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$ is the first negative term.**

A [CBSE OD Set-III, 2020]

Sol. Here, First term, $a = 20$

and Common difference, $d = \dfrac{77}{4} - 20 = -\dfrac{3}{4}$ 1

Let $t_n < 0$

$\because$ $t_n = a + (n-1)\, d$ ½

$\therefore$ $20 + (n-1)\left(-\dfrac{3}{4}\right) < 0$ ½

$\Rightarrow$ $80 - 3n + 3 < 0$

$\Rightarrow$ $83 - 3n < 0$

$\Rightarrow$ $n > \dfrac{83}{3}$

$\Rightarrow$ $n > 27.6$

$\Rightarrow$ $n = 28$

Hence, the first negative term is 28. 1

AI Q. 2. Find the middle term of the A.P. 7, 13, 19, ..., 247.

U [CBSE OD Set-III, 2020]

Sol. In this A.P., $a = 7$, $d = 13 - 7 = 6$ ½

and $t_n = 247$ ½

$\because \quad t_n = a + (n-1)d$

$\therefore \quad 247 = 7 + (n-1)6$

$\Rightarrow \quad 6(n-1) = 240$

$\Rightarrow \quad n - 1 = 40$

$\Rightarrow \quad n = 41$ 1

Hence,

$$\text{the middle term} = \frac{n+1}{2} = \frac{41+1}{2} = \frac{42}{2} = 21.$$ 1

Q. 3. For what value of n, are the n^{th} terms of two A.Ps 63, 65, 67,.... and 3, 10, 17,.... equal ?

C + A [CBSE Outside Delhi Set-III, 2017]

Topper Answer, 2017

Sol. Let a, d and A, D be the 1st term and common difference of the 2 A.Ps respectively.

n is same.

$a = 63$, $d = 2$

$A = 3$, $D = 7$

$a_n = A_n$

$\Rightarrow a + (n-1)d = A + (n-1)D$

$63 + (n-1)2 =$ [illegible] $(n-1)\,7$

$63 + 2n - 2 =$ [illegible] $7n - 7$

$61 + 2n =$ [illegible] -4

$65 =$ [illegible]

$13 = n$

$\therefore$ When n is 13, the n^{th} terms are equal

i.e., $a_{13} = A_{13}$. 3

Q. 4. If the 10^{th} term of an A.P. is 52 and the 17^{th} term is 20 more than the 13^{th} term, find A.P.

A [CBSE, Outside Delhi Set-I 2017]

Sol. $a_{10} = 52$

or, $a + 9d = 52$ ...(i) 1

Also $a_{17} - a_{13} = 20$

$a + 16d - (a + 12d) = 20$ ½

$4d = 20$

$d = 5$

Substituting, the value of d in (i), we get

$a = 7$ 1

Hence, A.P. = 7, 12, 17, 22 ..... ½

[CBSE Marking Scheme, 2017]

Q. 5. The ninth term of an A.P. is equal to seven times the second term and twelfth term exceeds five times the third term by 2. Find the first term and the common difference. A [CBSE SQP, 2016]

Sol. Let the first term of A.P. be a and common difference be d.

Given, $a_9 = 7a_2$

or, $a + 8d = 7(a + d)$ ...(i) ½

and $a_{12} = 5a_3 + 2$

Again, $a + 11d = 5(a + 2d) + 2$ ...(ii) 1

From (i), $a + 8d = 7a + 7d$

$-6a + d = 0$ ...(iii)

From (ii), $a + 11d = 5a + 10d + 2$

$-4a + d = 2$ ...(iv)

Subtracting (iv) from (iii), we get

$-2a = -2$

or, $a = 1$ 1

From (iii),

$-6 + d = 0$

$d = 6$ ½

Hence, first term = 1 and common difference = 6

[CBSE Marking Scheme, 2016]

Q. 6. The digits of a positive number of three digit number are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number. A [CBSE Delhi Set II, 2016]

Topper Answer, 2016

Sol. Let three digit of 3-digit no be – $a-d, a, a+d$.

Their sum = 15

$a-d+a+a+d = 15 \Rightarrow 3a = 15 \Rightarrow a = 5$

Required 3 digit no = $100(a-d)+10a+a+d$

$100a-100d+10a+a+d$

$111a-99d$

No. obtained by reversing digit = $100(a+d)+10a+a-d$

$100a+100d+10a+a-d$

$111a+99d$

A.T.Q. –

$111a+99d = 111a-99d-594$

$\Rightarrow 594 = 111a-99d-111a-99d$

$594 = -198d$

$\frac{-594}{198} = d$

$-3 = d.$

The no = $111a-99d$

$111\times5-99\times-3$

$555+297 = 852$

No $\Rightarrow$ 852

3

Q. 7. Divide 56 in four parts in A.P. such that the ratio of the product of their extremes (1st and 4th) to the product of middle (2nd and 3rd) is 5 : 6. U [Foreign Set I, 2016]

Sol. Let the four parts be

$a-3d, a-d, a+d$ and $a+3d$.

$\therefore a-3d+a-d+a+d+a+3d = 56$

or, $4a = 56$

$a = 14$ **1**

Hence, four parts are $14-3d$, $14-d$, $14+d$ and $14+3d$.

Now, according to question,

$$\frac{(14-3d)(14+3d)}{(14-d)(14+d)} = \frac{5}{6}$$

or, $$\frac{196-9d^2}{196-d^2} = \frac{5}{6}$$

or, $6(196-9d^2) = 5(196-d^2)$

or, $6\times196-54d^2 = 5\times196-5d^2$

or, $6\times196-5\times196 = 54d^2-5d^2$

or, $(6-5)\times196 = 49d^2$

or, $d^2 = \frac{196}{49} = 4$

or, $d = \pm 2$ **1**

$\therefore$ The four parts are

$\{14-3(\pm 2)\}, \{14-(\pm 2)\}$

Hence, first possible division will be 8, 12, 16 and 20. ½

and second possible division will be 20, 16, 12 and 8. ½

Q. 8. The p^{th}, q^{th} and r^{th} terms of an A.P. are a, b and c respectively. Show that $a(q-r)+b(r-p)+c(p-q) = 0$. U [Foreign Set II, 2016]

Sol. Let the first term be a' and the common difference be d.

$a = a'+(p-1)d$, $b = a'+(q-1)d$ and

$c = a'+(r-1)d$ **1½**

$a(q-r) = [a'+(p-1)d][q-r]$

$b(r-p) = [a'+(q-1)d][r-p]$

and $c(p-q) = [a'+(r-1)d][p-q]$ ½

$\therefore a(q-r)+b(r-p)+c(p-q) = a'[q-r+r-p+p-q]+d[(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)]$ ½

$= a'\times0+d[pq-pr+qr-pq+pr-qr+(-q+r-r+p-p+q)] = 0$ **Hence Proved.** ½

[CBSE Marking Scheme, 2016]

Q. 9. Prove that the n^{th} term of an A.P. can not be $n^2 + 1$. Justify your answer. [CBSE Board Term-2 2015]

Sol. Let n^{th} term of A.P.,

$$a_n = n^2 + 1$$

Putting the values of $n = 1, 2, 3, ...$, we get

$$a_1 = 1^2 + 1 = 2$$
$$a_2 = 2^2 + 1 = 5$$
$$a_3 = 3^2 + 1 = 10 \quad \mathbf{1}$$

The obtained sequence

$$= 2, 5, 10, 17, ...$$

Their common difference

$$= a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

or, $5 - 2 \neq 10 - 5 \neq 17 - 10$

$\therefore$ $3 \neq 5 \neq 7$ **1**

Since the common difference are not equal.
Hence, $n^2 + 1$ is not a form of n^{th} term of an A.P. **1**

[CBSE Marking Scheme, 2015]

Long Answer Type Questions — 5 marks each

AI **Q. 1. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last term to the product of two middle terms is 7 : 15. Find the numbers.** U [CBSE Delhi Set-I, 2020] [CBSE Delhi & OD, 2018]

Sol. Let the four consecutive terms of A.P. be $(a - 3d), (a - d), (a + d)$ and $(a + 3d)$. **1**

By given conditions

$a - 3d + a - d + d + a + 3d = 32$

$\Rightarrow$ $4a = 32$

$\Rightarrow$ $a = 8$ **1**

And $\dfrac{(a-3d)(a+3d)}{(a-d)(a+d)} = \dfrac{7}{15}$ **1**

$$\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$$

$$d^2 = 4$$

$$d = \pm 2 \quad \mathbf{1}$$

Hence, the numbers are 2, 6, 10 and 14 or 14, 10, 6 and 2. **1**

[CBSE Marking Scheme, 2018]

Q. 2. If m times the m^{th} term of an Arithmetic Progression is equal to n times its n^{th} term and $m \neq n$, show that the $(m + n)^{th}$ term of the A.P. is zero. [CBSE Term I, II, III, 2019]

Topper Answer, 2019

Sol.

Let the first term of given A.P. be 'a'
And, the common difference be 'd'.
and a_p dentotes p^{th} term.

given; $m(a_m) = n(a_n)$ $[m \neq n]$.

To show: $a_{(m+n)} = 0$

$m(a_m) = n(a_n)$

$\Rightarrow m[a + (m-1)d] = n[a + (n-1)d]$

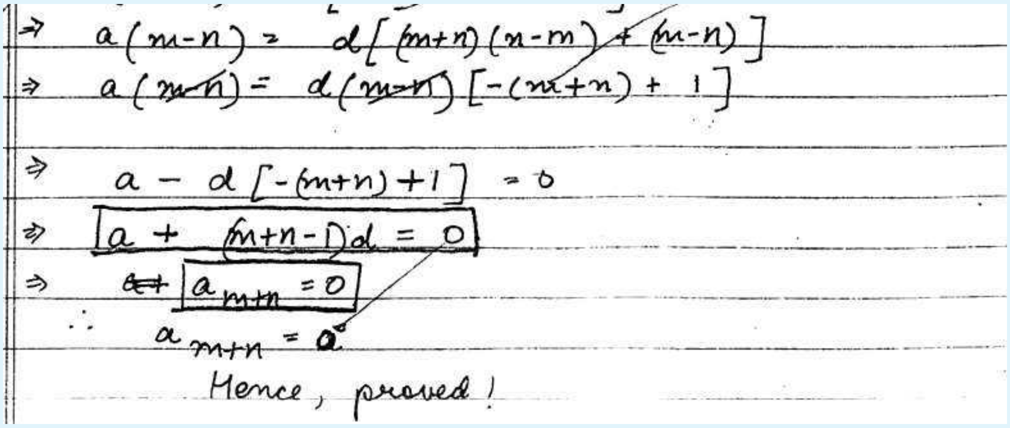
$\Rightarrow am + md(m-1) = an + nd(n-1)$

$\Rightarrow am - an = nd(n-1) - md(m-1)$

$\Rightarrow a(m-n) = d[n(n-1) - m(m-1)]$

$\Rightarrow a(m-n) = d[n^2 - n - m^2 + m]$

$\Rightarrow a(m-n) = d[n^2 - m^2 + m - n]$



$\Rightarrow a(m-n) = d[(m+n)(n-m) + (m-n)]$

$\Rightarrow a(m-n) = d(m-n)[-(m+n) + 1]$

$\Rightarrow a - d[-(m+n)+1] = 0$

$\Rightarrow a + (m+n-1)d = 0$

$\Rightarrow a_{m+n} = 0$

$\therefore a_{m+n} = 0$

Hence, proved!

5

Q. 3. An A.P. consists of 50 terms of which 3rd term is 12 and last term is 106. Find the 29th term.

U [CBSE SQP, 2018]

Sol. Given, $n = 50$, $a_3 = 12$ and $a_{50} = 106$ **1**

Then $a + 2d = 12$ **1**

and $a + 49d = 106$ **1**

On solving, we get $d = 2$ and $a = 8$ **1**

$$a_{29} = a + 28d$$
$$= 8 + 28 \times 2$$
$$= 64$$ **1**

[CBSE Marking Scheme, 2018]

Q. 4. The sum of three numbers in A.P. is 12 and sum of their cubes is 288. Find the numbers.

A [Delhi Set III, 2016]

Sol. Let the three numbers in A.P. be $a - d$, a and $a + d$.

Then, their sum *i.e.*, $3a = 12$ **1**

or, $a = 4$

Also, $(4-d)^3 + 4^3 + (4+d)^3 = 288$ **1**

or, $64 - 48d + 12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 = 288$

or, $24d^2 + 192 = 288$ **1**

or, $d^2 = 4$

$\therefore$ $d = \pm 2$ **1**

Hence, the numbers are 2, 4 and 6, or 6, 4 and 2. **1**

[CBSE Marking Scheme, 2016]

AI **Q. 5. Find the value of *a*, *b* and *c* such that the numbers *a*, 7, *b*, 23 and *c* are in A.P.**

U [CBSE Board Term-2, 2015]

Sol. Since, a, 7, b, 23 and c are in A.P.

Let the common difference be d

$\therefore$ $a + d = 7$ ...(i) ½

and $a + 3d = 23$ ...(ii) ½

From (i) and (ii), we get

$a = -1$ and $d = 8$ **1**

Again, $b = a + 2d$

$b = -1 + 2 \times 8$

or, $b = -1 + 16$

or, $b = 15$ **1**

$\therefore$ $c = a + 4d$

$= -1 + 4 \times 8$

$= -1 + 32$

$c = 31$ **1**

$\therefore a = -1, b = 15$ and $c = 31$ **1**

[CBSE Marking Scheme, 2015]

TOPIC - 2
Sum of *n* Terms of an Arithmetic Progression

Know the Formulae

- Sum of n terms of an A.P is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where, a is the first term, d is the common difference and n is the total number of terms.

- Sum of n terms of an A.P. when first and last term is given.

$$S_n = \frac{n}{2}[a + l]$$

where, a is the first term and l is the last term.

➢ The n^{th} term of an A.P is the difference of the sum of first n terms and the sum to first $(n - 1)$ terms of it. *i.e.*,

$$a_n = S_n - S_{n-1}.$$

How is it done on the GREENBOARD?

Q.1. Find the number of terms in the A.P 54, 51, 48, ... whose sum is 513. Also, give the reason of double answer.

Solution:

Step I: The given A.P. is 54, 51, 48, ...

Here $a = 54$, $d = 51 - 54 = -3$

Sum required is 513.

Step II: Applying the sum formula

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$513 = \frac{n}{2}[2 \times 54 + (n-1)(-3)]$$

$$1026 = n[108 - 3n + 3]$$

$$1026 = n[111 - 3n]$$

$$1026 = 111n - 3n^2$$

or, $$3n^2 - 111n + 1026 = 0$$

or, $$3[n^2 - 37n + 342] = 0$$

or, $$n^2 - 37n + 342 = 0$$

Step III: Factorizing the quadratic equation

$$n^2 - 19n - 18n + 342 = 0$$

$$n(n-19) - 18(n-19) = 0$$

or, $$(n-19)(n-18) = 0$$

or, $$n = 18 \text{ or } 19$$

Hence, the required number of terms will be 18 or 19.

19^{th} term of A.P. is zero hence double answers are correct.

Very Short Answer Type Questions — 1 mark each

Q. 1. Find the sum of the first 10 multiples of 6. A [CBSE Board Term, 2019]

Topper Answer, 2019

Sol. First 10 multiples of 6 forms AP → 6, 12, 18 ..... 60.
a l

Sum of 1st 10 multiples = $\frac{n}{2}[a + l]$

$= \frac{10}{2}[6 + 60]$

$= 330$ 1

Q. 2. If nth term of an A.P. is $(2n + 1)$, what is the sum of its first three terms ? A [CBSE SQP, 2018]

Sol. Since, $a_1 = 3$, $a_2 = 5$ and $a_3 = 7$ ½

$$S_3 = \frac{3}{2}(3 + 7) = 15$$ ½

[CBSE Marking Scheme, 2018]

Detailed Solution:

$\because \quad a_n = (2n + 1)$

$\therefore \quad a_1 = 2 \times 1 + 1 = 3$

$l = a_3 = 2 \times 3 + 1 = 7$ ½

Since, $S_n = \frac{n}{2}[a + l]$

Hence, $S_3 = \frac{3}{2}[3 + 7]$

$S_3 = 15.$ ½

Q. 3. If the first term of an A.P. is – 5 and the common difference is 2, then find the sum of the first 6 terms. R

Sol. In the given A.P., $a = -5$ and $d = 2$

Thus, $S_n = \frac{n}{2}[2a + (n-1)d]$ ½

$\therefore \quad S_6 = \frac{6}{2}[2 \times (-5) + (6-1) \times 2]$

$= 3(-10 + 10)$

$= 0.$ ½

Short Answer Type Questions-I — 2 marks each

AI **Q. 1. Find the sum of first 20 terms of the following A.P.:**

1, 4, 7, 10, ...

A [CBSE Delhi Set-II, 2020]

Sol. Given A.P.: 1, 4, ,7, 10, ...

Here, $a = 1, d = 4 - 1 = 3$ and $n = 20$ ½

$\therefore$ The sum of first 20 terms,

$S_{20} = \frac{n}{2}[2a + (n-1)d]$ ½

$= \frac{20}{2}[2 \times 1 + (20-1)3]$

$= 10(2 + 57)$

$= 10 \times 59$

$= 590.$ 1

AI **Q. 2. The sum of the first 7 terms of an A.P. is 63 and that of its next 7 terms is 161. Find the A.P. .**

A [CBSE Delhi Set-III, 2020]

Sol. Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

Given, $S_7 = 63$

So, $S_7 = \frac{7}{2}[2a + 6d]$

$= 63$

or, $2a + 6d = 18$ ...(i) ½

Now, sum of 14 terms is:

$S_{14} = S_{\text{first 7 terms}} + S_{\text{next 7 terms}}$

$= 63 + 161 = 224$

$\therefore \quad \frac{14}{2}[2a + 13d] = 224$

$\Rightarrow \quad 2a + 13d = 32$ ...(ii) ½

On subtracting (i) from (ii), we get

$(2a + 13d) - (2a + 6d) = 32 - 18$

$\Rightarrow \quad 7d = 14$

$\Rightarrow \quad d = 2$

Putting the value of d in (i), we get

$a = 3$ ½

Hence, the A.P. will be: 3, 5, 7, 9, ... ½

AI **Q. 3. If S_n, the sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$. Find the n^{th} term.**

A [CBSE Delhi Set-I, 2019]

Sol. $a_1 = S_1 = 3 - 4 = -1$ ½

$a_2 = S_2 - S_1$

$= [3(2)^2 - 4(2)] - (-1) = 5$ ½

$\therefore \quad d = a_2 - a_1 = 6$ ½

Hence $a_n = -1 + (n-1) \times 6 = 6n - 7$ ½

Alternate method:

$S_n = 3n^2 - 4n$

$\therefore \quad S_{n-1} = 3(n-1)^2 - 4(n-1)$

$= 3n^2 - 10n + 7$ **1**

Hence $a_n = S_n - S_{n-1}$ ½

$= (3n^2 - 4n) - (3n^2 - 10n + 7)$

$= 6n - 7$ ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, $S_n = 3n^2 - 4n$

Put $n = 1, S_1 = 3 \times 1^2 - 4 \times 1 = -1$ ½

So, sum of first term of A.P. is – 1.

But sum of first term will be the first term,

$\therefore$ First team, $a_1 = -1$

Put $n = 2$, $S_2 = 3 \times 2^2 - 4 \times 2 = 4$ ½

$\therefore$ Sum of first two terms is 4.

$\therefore$ First term + Second term = 4

$\therefore \quad -1 + a_2 = 4$

$\Rightarrow \quad a_2 = 5$ ½

Hence, Common difference, $d = a_2 - a_1 = 5 - (-1) = 6$

$\therefore \quad n^{th}$ term, $a_n = a_1 + (n-1)d$

i.e., $a_n = -1 + (n-1)6$

$\Rightarrow \quad a_n = 6n - 7$

Therefore, n^{th} term is $6n - 7$. ½

COMMONLY MADE ERROR

- Some students do not know the basic concepts of arithmetic progression. Many students try to solve with wrong method.

ANSWERING TIP

- Learn the concept of Arithmetic progression with different examples.

Q. 4. Find the sum of first 8 multiples of 3. A [Delhi/OD 2018] [Delhi Comptt. Set-I, 2017]

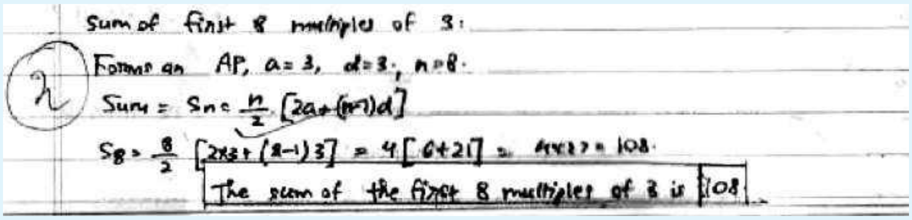
Sol. Here,

$$S = 3 + 6 + 9 + 12 + ... + 24$$
$$= 3(1 + 2 + 3 + ... + 8) \quad \mathbf{1}$$
$$= 3 \times \frac{8 \times 9}{2}$$
$$= 108 \quad \mathbf{1}$$

[CBSE Marking Scheme, 2018]

Detailed Solution:

Topper Answer, 2019


Sum of first 8 multiples of 3:
Forms an AP, a = 3, d = 3, n = 8.
Sum = Sn = n/2 [2a+(n−1)d]
S8 = 8/2 [2×3+(8−1)3] = 4[6+21] = 4×27 = 108.
The sum of the first 8 multiples of 3 is 108

1

Q. 5. How many terms of the A.P. $-6, \frac{-11}{2}, -5, -\frac{9}{2}$..... are needed to give their sum zero.
A [CBSE Delhi comptt. Set-III, 2017]
[CBSE Delhi Set-III, 2016]

Sol. Given $a = -6$ and $d = -\frac{11}{2} - (-6) = \frac{1}{2}$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

Let sum of n terms be zero.

$\therefore$ $S_n = 0$

or, $\frac{n}{2}\left[2 \times -6 + (n-1)\frac{1}{2}\right] = 0$ ½

or, $\frac{n}{2}\left[-12 + \frac{n}{2} - \frac{1}{2}\right] = 0$

or, $\frac{n}{2}\left[\frac{n}{2} - \frac{25}{2}\right] = 0$

or, $n^2 - 25n = 0$ **1+½**

$n(n - 25) = 0$

$n = 25$ as $n \neq 0$

Hence, terms are needed = 25.

Q. 6. In an A.P. of 50 terms, the sum of the first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the A.P. A [Foreign Set-III, 2017]

Sol. Given, $S_{10} = 210$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

or, $\frac{10}{2}(2a + 9d) = 210$ ½

or, $2a + 9d = 42$ ...(i)

Since, $a_{36} = a + 35d$

and $a_{50} = a + 49d$

Hence,

Sum of last 15 terms $= \frac{15}{2}(a + 35d + a + 49d)$

or, $\frac{15}{2}(2a + 84d) = 2565$ ½

or, $a + 42d = 171$ ...(ii) ½

On solving (i) and (ii), we get

$a = 3$ and $d = 4$ ½

Hence, given A.P. is 3, 7, 11 ....

[CBSE Marking Scheme, 2017]

Q. 7. Reshma wanted to save at least ₹ 6,500 for sending her daughter to school next year (after 12 months). She saved ₹ 450 in the first month and raised her savings by ₹ 20 every next month. How much will she be able to save in next 12 months ? Will she be able to send her daughter to the school next year ?
C [Foreign Set-I, II, III, 2016]
[CBSE Delhi Term-II Set-I, II, III, 2015]

Sol. Here $a = ₹\ 450$, $d = ₹\ 20$, $n = 12$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2 \times 450 + 11 \times 20]$$
$$= 6[1120]$$
$$= 6720 > 6500 \quad \mathbf{2}$$

$\therefore$ Reshma will be able to send her daughter to school. [CBSE Marking Scheme, 2016]

Q. 8. In an A.P., if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the A.P., where S_n denotes the sum of first n terms.
A [CBSE Board, Term-2 2015]

Sol. $S_n = \frac{n}{2}[2a+(n-1)d]$

Given, $S_5 + S_7 = 167$

Hence, $\frac{5}{2}(2a+4d)+\frac{7}{2}(2a+6d) = 167$

or, $24a + 62d = 334$

or $12a + 31d = 167$ ....(i) ½

Given, $S_{10} = 235$

or, $5(2a + 9d) = 235$

or $2a + 9d = 47$ ...(ii) ½

Solving (i) and (ii), wet get

$a = 1$ and $d = 5$ ½

Hence A.P. = 1, 6, 11, ... ½

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-II — 3 marks each

AI **Q. 1. Show that the sum of all terms of an A.P. whose first term is *a*, the second term is *b* and the last term is *c* is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$.**

A [CBSE OD Set-I, 2020]

Sol. Given, first term, $A = a$

and second term $= b$

$\Rightarrow$ common difference, $d = b - a$

Last term, $l = c$

$\Rightarrow A + (n-1)d = c$

[By using, $l = a + (n-1)d$] **1**

$\Rightarrow a + (n-1)d = c$

$a + (n-1)(b-a) = c$

$\Rightarrow (b-a)(n-1) = c - a$

$\Rightarrow n - 1 = \frac{c-a}{b-a}$

$\Rightarrow n = \frac{c-a}{b-a}+1$

$= \frac{c-a+b-a}{b-a}$

$\Rightarrow n = \frac{b+c-2a}{b-a}$ **1**

Now sum $= \frac{n}{2}[A + l]$

$= \frac{(b+c-2a)}{2(b-a)}[a+c]$

$= \frac{(a+c)(b+c-2a)}{2(b-a)}$ **1**

Hence Proved.

AI **Q. 2. Solve the equation: 1 + 4 + 7 + 10 + ... + *x* = 287.** A [CBSE Delhi OD Set-I, 2020]

Sol. Given, $a = 1$ and $d = 4 - 1 = 3$ ½

Let number of terms is the series be n, then

$S_n = \frac{n}{2}[2a + (n-1)d]$ ½

$\Rightarrow \frac{n}{2}[2 \times 1 + (n-1)3] = 287$ ½

$\Rightarrow \frac{n}{2}[2 + 3n - 3] = 287$

$\Rightarrow 3n^2 - n - 574 = 0$ ½

$\Rightarrow 3n^2 - 42n + 41n - 574 = 0$

$\Rightarrow 3n(n-14) + 41(n-14) = 0$

$\Rightarrow (n-14)(3n+41) = 0$

Either $n = 14$ or $n = -\frac{41}{3}$, it is not possible.

Thus 14^{th} term is x

$\therefore a + (n-1)d = x$

$\Rightarrow x = 1 + 13 \times 3$

$= 40.$ **1**

AI **Q. 3. If in an A.P., the sum of first *m* terms is *n* and the sum of its first *n* terms is *m*, then prove that the sum of its first (*m* + *n*) terms is –(*m* + *n*).**

A [CBSE OD Set-II, 2020]

Sol. Let 1^{st} term of series be a and common difference be d, then

$S_m = n$ (given)

$\Rightarrow \frac{m}{2}[2a + (m-1)d] = n$ ½

$\Rightarrow m[2a + (m-1)d] = 2n$ ...(i) ½

and $S_n = m$ (given)

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = m$

$\Rightarrow n[2a + (n-1)d] = 2m$ ...(ii) ½

On subtracting,

$2(n-m) = 2a(m-n) + d[m^2 - n^2 - (m-n)]$

$\Rightarrow 2(n-m) = 2a(m-n) + d[(m-n)]$

$[-(m-n)-(m-n)]$

$\Rightarrow 2(n-m) = (m-n)[2a + d(m+n-1)]$

$\Rightarrow -2 = 2a + d(m+n-1)$ ½

Now, $S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d]$

$= \frac{m+n}{2}(-2)$

$= -(m+n)$ **Hence Proved. 1**

Q. 4. Find the sum of all 11 terms of an A.P. whose middle term is 30. A [CBSE OD Set-II, 2020]

Sol. In an A.P. with 11 terms,

middle term $= \frac{11+1}{2}$ term

$= 6^{th}$ term **1**

Now, sixth term *i.e.*, $a_6 = a + (6-1)d$

i.e., $a + 5d = 30$ ...(i)

[$\because$ middle term *i.e.*, $a_6 = 30$ (given)] **1**

Now, the sum of 11 terms,

$$S_{11} = \frac{11}{2}[2a + (11-1)d]$$
$$= \frac{11}{2}[2a + 10d]$$
$$= \frac{11}{2} \times 2[a + 5d]$$
$$= 11 \times 30 \qquad \text{[from (i)]}$$
$$= 330. \qquad 1$$

Q. 5. If the sum of first m terms of an A.P. is the same as the sum of its first n terms, show that the sum of its first $(m + n)$ terms is zero. A [CBSE SQP, 2020]

Sol. $S_m = S_n$

$\Rightarrow \frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n - 1d]$ 1

$\Rightarrow 2a(m-n) + d(m^2 - m - n^2 + n) = 0$ 1

$\Rightarrow (m-n)[2a + (m+n-1)d] = 0$ 1

or $S_{m+n} = 0$

[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

Sum of first m terms = Sum of first n terms

$\Rightarrow S_m = S_n$ ½

$\frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$ ½

$$m[2a + (m-1)d] = n[2a + (n-1)d]$$
$$m[2a + (m-1)d] - n[2a + (n-1)d] = 0$$
$$2a(m-n) + [m(m-1) - n(n-1)]d = 0 \qquad 1$$
$$2a(m-n) + [m^2 - m - n^2 + n]d = 0$$
$$2a(m-n) + [(m-n)(m+n) - (m-n)]d = 0$$
$$(m-n)[2a + (m+n-1)d] = 0$$

Here, $(m - n)$ is not equal to zero.

So, $[2a + (m + n - 1)d] = 0$

Hence, $S_{m+n} = 0.$ 1

Q. 6. If the sum of first four terms of an A.P. is 40 and that of first 14 terms is 280. Find the sum of its first n terms. A [CBSE Delhi Set-I, 2019]

Sol. $S_4 = 40 \Rightarrow 2(2a + 3d) = 40 \Rightarrow 2a + 3d = 20$ ½

$S_{14} = 280 \Rightarrow 7(2a + 13d) = 280 \Rightarrow 2a + 13d = 40$ ½

Solving to get $d = 2$ ½

and $a = 7$ ½

$\therefore S_n = \frac{n}{2}[14 + (n-1)2]$ ½

$= n(n + 6)$ or $(n^2 + 6n)$ ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Since,

Sum of n terms of an A.P.,

$S_n = \frac{n}{2}[2a + (n-1)d]$ ½

[a be the first term and d be the common difference]

According to question, $S_4 = 40$

$\Rightarrow \frac{4}{2}[2a + (4-1)d] = 40$ ½

$\Rightarrow 2[2a+3d] = 40$

$\Rightarrow 2a + 3d = 20$ ...(i) ½

and $S_{14} = 280$

$\Rightarrow \frac{14}{2}[2a + (14-1)d] = 280$ ½

$\Rightarrow 7(2a + 13d) = 280$

$\Rightarrow 2a + 13d = 40$ ...(ii) ½

Solving eq. (i) and (ii), we get

$a = 7$ and $d = 2$

$\therefore S_n = \frac{n}{2}[2 \times 7 + (n-1)2]$

$= \frac{n}{2}[14 + 2n - 2]$

$= \frac{n}{2}(12 + 2n)$

$= 6n + n^2$

Hence, Sum of n terms $= 6n + n^2$. ½

Topper Answer, 2019

Sol.

$a = 9$, $d = 8$, $S_n = 636.$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$636 = \frac{n}{2}[18 + (n-1)8]$

$636 = n[9 + (n-1)4]$

$636 = n(9 + 4n - 4)$

$636 = n(5 + 4n)$

$636 = 5n + 4n^2$

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

$n(4n + 5$

$4n^2 - 48n + 53n - 636 = 0$

$4n(n-12) + 53(n-12) = 0$

$(4n+53)(n-12) = 0$

$\therefore n = \frac{-53}{4}$ or 12.

as n is a natural number, $\boxed{n = 12}$

$\therefore$ 12 terms are required to give sum 636. **3**

Q. 7. How many terms of an A.P. 9, 17, 25, ... must be taken to give a sum of 636 ? A [CBSE OD Set-III, 2017]

Topper Answer, 2017

Sol. $a = 9$, $d = 8$, $S_n = 636$.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$636 = \frac{n}{2}[18 + (n-1)8]$

$636 = n[9 + (n-1)4]$

$636 = n(9 + 4n - 4)$

$636 = n(5 + 4n)$

$636 = 5n + 4n^2$

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

$4n^2 - 48n + 53n - 636 = 0$

$4n(n-12) + 53(n-12) = 0$

$(4n+53)(n-12) = 0$

$\therefore n = \frac{-53}{4}$ or 12.

as n is a natural number, $\boxed{n = 12}$

$\therefore$ 12 terms are required to give sum 636. **3**

AI **Q. 8. Find the sum of n terms of the series**

$$\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+......$$

A [CBSE Delhi Set-I, II, III, 2017]

Sol. Let sum of n term be S_n

$$\therefore S_n = \left[4-\frac{1}{n}\right]+\left[4-\frac{2}{n}\right]+\left[4-\frac{3}{n}\right]+......$$

up to n terms **1**

or, (4 + 4 + 4 + ... up to n terms)

$$-\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+........ \text{ up to } n \text{ terms}\right)$$

or, (4 + 4 + 4 + ... up to n terms)

$$-\frac{1}{n}(1 + 2 + 3 + ... \text{ up to } n \text{ terms})$$

or, (4 + 4 + 4 + ... up to n terms)

$$-\frac{1}{n}(1 + 2 + 3 + ... \text{ up to } n \text{ terms})$$

or, $4n - \frac{1}{n} \times \frac{n(n+1)}{2}$ **1½**

or, $4n - \frac{n+1}{2} = \frac{7n-1}{2}$

Hence, sum of n terms $= \frac{7n-1}{2}$ **½**

[CBSE Marking Scheme, 2017]

Q. 9. If the sum of the first 14 terms of an A.P. is 1050 and its first term is 10, find its 20th term.

A [CBSE OD Comptt. Set-III, 2017]

Sol. Given, $a = 10$, and $S_{14} = 1050$

Let the common difference of the A.P. be d. ½

Since, $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\therefore \quad S_{14} = \frac{14}{2}[2\times10+(14-1)d]$$

$$= 1050 \quad ½$$

$$20 + 13d = \frac{1050}{7} = 150$$

$$13d = 130$$

$$d = \frac{130}{13} = 10 \quad 1$$

$$a_n = a + (n-1)d$$

$$a_{20} = 10 + 19 \times 10 = 200 \quad 1$$

Hence, $a_{20} = 200$.

[CBSE Marking Scheme, 2017]

Q. 10. Find the sum of all odd numbers between 0 and 50.

A [Delhi Comptt. Set-III, 2017]

Sol. Given, $1 + 3 + 5 + 7 + ... + 49$

Let, total odd numbers of terms be n. 1

$$a_n = 1 + (n-1) \times 2 = 49$$

$$(n-1) \times 2 = 49 - 1 = 48$$

$$n - 1 = 24$$

$$n = 24 + 1 = 25 \quad 1$$

$$S_{25} = \frac{25}{2}(1+49)$$

$$= 25 \times 25$$

$$= 625$$

Hence, sum of odd numbers between 0 and 50 $= 625$ 1

[CBSE Marking Scheme, 2017]

AI **Q. 11. If m^{th} term of A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, find the sum of first mn terms.** A [CBSE Set-I, II, 2017]

Sol. Let first term of given A.P. be a and common difference be d.

$$\therefore \quad a_m = a + (m-1)d = \frac{1}{n} \quad ...(i) \ ½$$

and $$a_n = a + (n-1)d = \frac{1}{m} \quad ...(ii) \ ½$$

On subtracting (ii) from (i) we get

$$(m-n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn} \quad 1$$

or, $$d = \frac{1}{mn}$$

and $$a = \frac{1}{mn}$$

Now $$S_{mn} = \frac{mn}{2}\left(2\cdot\frac{1}{mn} + (mn-1)\frac{1}{mn}\right)$$

$$= \frac{mn}{2}\left(\frac{2}{mn} + \frac{mn}{mn} - \frac{1}{mn}\right)$$

$$S_{mn} = \frac{mn}{2}\left[\frac{1}{mn} + 1\right]$$

$$= \frac{1}{2}[mn+1]$$

Hence, the sum of first mn terms $= \frac{1}{2}[mn+1]$. 1

[CBSE Marking Scheme, 2017]

Q. 12. Find the sum of all two digit natural numbers which are divisible by 4.

A [Delhi Comptt. Set-II, 2017]

Sol. First two digit multiple of 4 is 12 and last is 96

So, $a = 12$, $d = 4$ and $l = 96$

Let n^{th} term be last term $= 96$ 1

$$\therefore \quad a_n = a + (n-1)d = l$$

$$12 + (n-1)4 = 96$$

$$n - 1 = 21$$

$$n = 21 + 1 = 22 \quad 1$$

Now, $$S_{22} = \frac{22}{2}[12+96]$$

$$= 11 \times 108$$

$$= 1188 \quad 1$$

[CBSE Marking Scheme, 2017]

Q. 13. Find the sum of the following series:

5 + (– 41) + 9 + (– 39) + 13 + (– 37) + 17 + ... + (– 5) + 81 + (– 3) A [Foreign Set-I, 2017]

Sol. The series can be written as

(5 + 9 + 13 + ... + 81) + [(– 41) + (– 39) + (– 37) + (– 35) ... (– 5) + (– 3)]

For the series (5 + 9 + 13 + ... + 81) ½

$$a = 5$$

$$d = 4$$

and $$a_n = 81$$

Then, $$a_n = 5 + (n-1)4$$

$$= 81$$

or, $$(n-1)4 = 76 \quad ½$$

$$n = 20$$

$$S_n = \frac{20}{2}(5+81)$$

$$= 860$$

For series (– 41) + (– 39) + (– 37) + ... + (– 5) + (– 3)

$$a_n = -3 \quad ½$$

$$a = -41$$

$$d = 2$$

Then, $$a_n = -41 + (n-1)(2)$$

$$\therefore \quad n = 20$$

$$S_n = \frac{20}{2}[-41+(-3)]$$

$$= -440 \quad ½$$

Hence, the Sum of the series $= 860 - 440$

$$= 420 \quad 1$$

[CBSE Marking Scheme, 2017]

Q. 14. The sum of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each A.P. is 1 and common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$. A [OD Set III, 2016]

Sol. Since, $S_1 = 1 + 2 + 3 + ... + n.$

$S_2 = 1 + 3 + 5 + ...\text{upto } n \text{ terms}$

and $S_3 = 1 + 4 + 7 + ...\text{upto } n \text{ terms}$

or, $S_1 = \frac{n(n+1)}{2}$ ½

Also, $S_2 = \frac{n}{2}[2 \times 1 + (n-1)2]$

$= \frac{n}{2}[2n] = n^2$ ½

and $S_3 = \frac{n}{2}[2 \times 1 + (n-1)3]$

$= \frac{n(3n-1)}{2}$ ½

Now, $S_1 + S_3 = \frac{n(n+1)}{2} + \frac{n(3n-1)}{2}$ ½

$= \frac{n[n+1+3n-1]}{2}$

$= \frac{n[4n]}{2}$

$= 2n^2$

$= 2S_2$ **Hence Proved. 1**

[CBSE Marking Scheme, 2016]

Q. 15. If the sum of the first n terms of an A.P. is $\frac{1}{2}[3n^2 + 7n]$, then find its n^{th} term. Hence write its 20^{th} term.

A [CBSE Board Term-2, Set-II 2015]

[CBSE SQP-2016]

Sol. $S_n = \frac{1}{2}[3n^2 + 7n]$

$S_1 = \frac{1}{2}[3 \times (1)^2 + 7(1)]$

$= \frac{1}{2}[3 + 7]$

$= \frac{1}{2} \times 10 = 5$ ½

$S_2 = \frac{1}{2}[3(2)^2 + 7 \times 2]$

$= \frac{1}{2}[12 + 14]$

$= \frac{1}{2} \times 26$

$= 13$ ½

$a_1 = S_1 = 5$ ½

$a_2 = S_2 - S_1 = 13 - 5 = 8$ ½

$d = a_2 - a_1 = 8 - 5 = 3$ ½

Now, A.P. is 5, 8, 11, ...

n^{th} term, $a_n = a + (n-1)d$

$= 5 + (n-1)3$

$= 3n + 2$

Hence, $a_{20} = 3 \times 20 + 2$

$a_{20} = 62$ ½

[CBSE Marking Scheme, 2015]

Q. 16. Aditi required ₹ 2500 after 12 weeks to send her daughter to school. She saved ₹ 100 in the first week and increased her weekly saving by ₹ 20 every week. Find whether she will be able to send her daughter after 12 weeks.

C [CBSE Board Term-2, Set-I, II, III, 2015]

Sol. Here, required money is ₹ 2500

a = saving in 1st week = ₹ 100

d = difference in weekly saving = ₹ 20

A.P. formed by saving,

According to the question,

Sequence is 100, 120, 140, ... upto 12 terms

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{12} = \frac{12}{2}[2 \times 100 + (12-1) \times 20]$

or, $= 6[200 + 11 \times 20]$

or, $= 6[200 + 220]$

or, $= 6 \times 420$

$= 2520$ **3**

She will be able to send her daughter to school after 12 weeks. [CBSE Marking Scheme, 2015]

Q. 17. If S_n denotes, the sum of the first n terms of an A.P. prove that $S_{12} = 3(S_8 - S_4)$.

A [CBSE Delhi Board, Set-I, 2015]

Sol. Let a be the first term and d be the common difference.

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = 6[2a + 11d]$

$= 12a + 66d$ ...(i) **1**

$S_8 = 4[2a + 7d]$

$= 8a + 28d$ ½

and $S_4 = 2[2a + 3d]$

$= 4a + 6d$ ½

Then, $3(S_8 - S_4) = 3[(8a + 28d) - (4a + 6d)]$

$= 3[4a + 22d]$

$= 12a + 66d$

From equation (i) and (ii), $S_{12} = 3(S_8 - S_9)$ **1**

[CBSE Marking Scheme, 2015]

Q. 18. The 14^{th} term of an A.P. is twice its 8^{th} term. If the 6^{th} term is – 8, then find the sum of its first 20 terms. A [CBSE OD Set-I, 2015]

[Foreign Set-I, II, 2015]

Sol. Let first term be a and common difference be d.

Here, $a_{14} = 2a_8$

or, $a + 13d = 2(a + 7d)$

$a + 13d = 2a + 14d$

$a = -d$ ...(i) ½

Again, $a_6 = -8$

or, $a + 5d = -8$ ...(ii) ½

Solving (i) and (ii), we get

$a = 2, d = -2$ ½

$S_{20} = \frac{20}{2}[2\times2+(20-1)(-2)]$ ½

$= 10[4 + 19 \times (-2)]$

$= 10(4 - 38)$

$= 10 \times (-34) = -340$ **1**

[CBSE Marking Scheme, 2015]

Long Answer Type Questions — 5 marks each

AI Q. 1. Solve: 1 + 4 + 7 + 10 + ... + x = 287.

A [CBSE Delhi Set-I, 2020]

Sol. See the solution of Q. 2. from Short Answer Type Question-II.

AI Q. 2. The first term of an A.P. is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of the A.P..

[CBSE Delhi Set-II, 2019]

Sol. Here $a = 3$, $a_n = 83$ and $S_n = 903$ **1**

Therefore $83 = 3 + (n-1)d$

$\Rightarrow (n-1)d = 80$ ...(i) **1**

Also $903 = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}(6 + 80)$

$= 43n$ (using (i)) **1+½**

$\Rightarrow n = 21$

and $d = 4$ **1+½**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given:

First term, $a = 3$

Last term, $a_n = 83$

Sum of n terms, $S_n = 903$ **1**

Since, $S_n = \frac{n}{2}(a + a_n)$ **1**

$\Rightarrow 903 = \frac{n}{2}(3 + 83)$

$\Rightarrow 1806 = 86n$

$\Rightarrow n = \frac{1806}{86}$

$\Rightarrow n = 21$ **1**

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$ **1**

$\Rightarrow 903 = \frac{21}{2}[2 \times 3 + (21-1)d]$

$\Rightarrow 1806 = 21(6 + 20d)$

$\Rightarrow 6 + 20d = 86$

$\Rightarrow 20d = 80$

$\Rightarrow d = 4$

Hence, the common difference is 4. **1**

COMMONLY MADE ERROR

- Some students fail to find the value of n as they get confused between the n[th] term and last term.

ANSWERING TIP

- Understand the formulae related to given condition and use them to solve the problems.

AI Q. 3. If the ratio of the sum of the first n terms of two A.Ps is $(7n + 1) : (4n + 27)$, then find the ratio of their 9th terms.

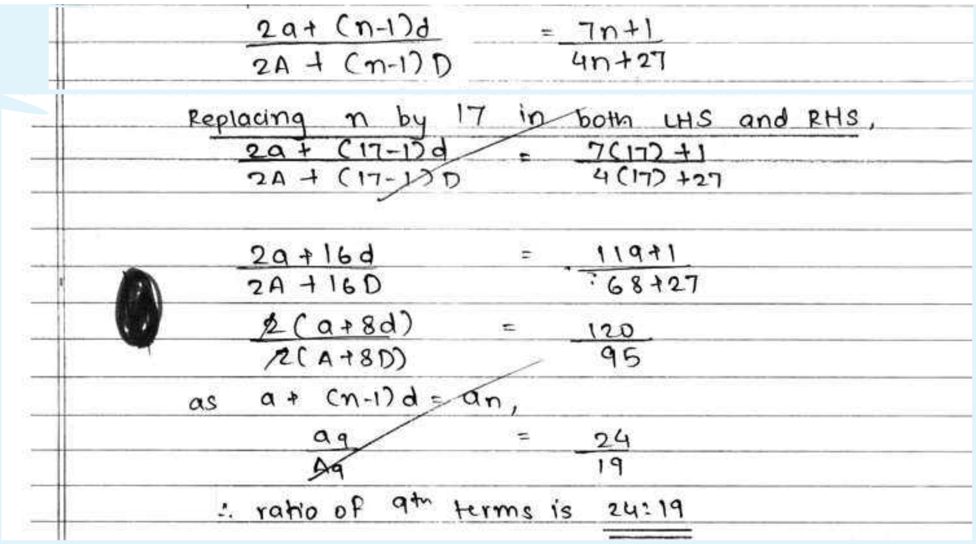
A [CBSE OD Set III 2017] [CBSE OD Set-I, 2016]

Topper Answer, 2017

Let a, d and A, D be the 1st term and common difference of the 2 APs respectively.

Then,

$$\frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2A + (n-1)D]} = \frac{7n+1}{4n+27}$$


$$\frac{2a + (n-1)d}{2A + (n-1)D} = \frac{7n+1}{4n+27}$$

Replacing n by 17 in both LHS and RHS,

$$\frac{2a + (17-1)d}{2A + (17-1)D} = \frac{7(17)+1}{4(17)+27}$$

$$\frac{2a+16d}{2A+16D} = \frac{119+1}{68+27}$$

$$\frac{2(a+8d)}{2(A+8D)} = \frac{120}{95}$$

as $a + (n-1)d = a_n$,

$$\frac{a_9}{A_9} = \frac{24}{19}$$

∴ ratio of 9th terms is 24 : 19

5

Q. 4. The ratio of the sums of first m and first n terms of an A.P. is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is $(2m - 1) : (2n - 1)$.

[CBSE Delhi Set-I, 2017]

Sol. Let first term of given A.P. be a and common difference be d also sum of first m and first n terms be S_m and S_n respectively.

$$\therefore \quad \frac{S_m}{S_n} = \frac{m^2}{n^2} \qquad 1$$

$$\text{or,} \quad \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2} \qquad 1$$

$$\text{or,} \quad \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m^2}{n^2} \times \frac{n}{m} = \frac{m}{n} \qquad 1$$

$$\text{or,} \quad m(2a + (n-1)d) = n[2a + (m-1)d] \qquad 1$$

$$d = 2a$$

$$\text{Now} \quad \frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d}$$

$$= \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a}$$

$$\text{or,} \quad \frac{a+2ma-2a}{a+2na-2a} = \frac{2ma-a}{2na-a} \qquad 1$$

$$= \frac{a(2m-1)}{a(2n-1)}$$

$$= 2m - 1 : 2n - 1$$

Hence Proved.

AI Q. 5. If the p^{th} term of an A.P. is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$. Prove that the sum of first pq term of the A.P. is $\left[\frac{pq+1}{2}\right]$.

[CBSE Delhi Set-III, 2017]

Sol. Try yourself similar to Q.No. 11 of VSATQ-II.

Q. 6. If the ratio of the 11^{th} term of an A.P. to its 18^{th} term is 2 : 3, find the ratio of the sum of the first five term to the sum of its first 10 terms.

[Delhi Comptt. Set-I, II, III, 2017]

Sol. Since, $$\frac{a_{11}}{a_{18}} = \frac{a+10d}{a+17d} = \frac{2}{3}$$

$$\text{or,} \quad 2(a + 17d) = 3(a + 10d) \qquad 1$$

$$a = 4d \qquad ...(i)$$

$$\text{Now,} \quad \frac{S_5}{S_{10}} = \frac{\frac{5}{2}(2a+4d)}{\frac{10}{2}[2a+9d]} \qquad 1$$

Putting the value of $a = 4d$, we get 1

$$\text{or,} \quad \frac{S_5}{S_{10}} = \frac{\frac{5}{2}(8d+4d)}{5(8d+9d)} \qquad 1$$

$$\frac{12d}{34d} = \frac{6}{17} \qquad 1$$

Hence, $S_5 : S_{10} = 6 : 17$.

Q. 7. An A.P. consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three terms is 429. Find the A.P. [CBSE SQP, 2017]

Sol. Let the middle most terms of the A.P. be $(a - d)$, a and $(a + d)$.

Given, $a - d + a + a + d = 225$

or, $3a = 225$ 1

or, $a = 75$ 1

and the middle term $= \frac{37+1}{2} = 19^{th}$ term

∴ A.P. is

$(a - 18d)$, ..., $(a - 2d)$, $(a - d)$, a, $(a + d)$, $(a + 2d)$, ... , $(a + 18d)$ 1

Sum of last three terms

$(a + 18d) + (a + 17d) + (a + 16d) = 429$

or, $3a + 51d = 429$

or, $225 + 51d = 429$ or, $d = 4$ 1

First term, $a_1 = a - 18d = 75 - 18 \times 4 = 3$.

$a_2 = 3 + 4 = 7$

Hence, A.P. = 3, 7, 11, ... , 147. 1

Q. 8. The minimum age of children to be eligible to participate in a painting competition is 8 years. It is observed that the age of youngest boy was 8 years and the ages of rest of participants are having a common difference of 4 months. If the sum of ages

of all the participants is 168 years, find the age of eldest participant in the painting competition.

C [CBSE SQP, 2016]

Sol. Here, $a = 8$, $d = 4$ months $= \frac{1}{3}$ years and

$S_n = 168$ **1**

Since $S_n = \frac{n}{2}[2a+(n-1)d]$

Hence, $168 = \frac{n}{2}\left[2(8)+(n-1)\frac{1}{3}\right]$ **1**

$n^2 + 47n - 1008 = 0$ **1**

or, $n^2 + 63n - 16n - 1008 = 0$

or, $(n-16)(n+63) = 0$

or, $n = 16$ or $n = -63$

$n = 16$

(n cannot be negative So – 63 rejected) **1**

Thus, the age of the eldest participant $= a + 15d$ $= 13$ years [CBSE Marking Scheme, 2016] **1**

Q. 9. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after, the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief.

C [CBSE Delhi Set I, II, 2016]

Sol. Let total time to catch the thief be n minutes.

Then, total distance covered by thief = $(100n)$ metres ½

Total distances to be covered by policeman = 100 + 110 + 120 + ... + $(n-1)$ terms **1**

$\therefore \quad 100n = \frac{n-1}{2}[200 + (n-2)10]$ **1**

$n^2 - 3n - 18 = 0$ ½

$(n-6)(n+3) = 0$ ½

or, $n = 6$ ½

Policeman takes 6 minutes to catch the thief. **1**

[CBSE Marking Scheme, 2016]

Visual Case Based Questions — 4 marks each

Note: Attempt any four sub parts from each question. Each sub part carries 1 mark

Q. 1. India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year. [CBSE QB, 2021]

Based on the above information, answer the following questions:

(i) Find the production during first year.

Sol. ₹ 5000

Explanation: $a_6 = 16,000$

$a + (n+1)d = 16,000$

$a + (6-1)d = 16,000$

$a + 5d = 16,000$ ...(i)

$a_9 = 22,600$

$a + (n-1)d = 22,600$

$a + (9-1)d = 22,600$

$a + 8d = 22,600$ ...(ii)

Solving equation (i) and (ii)

$a + 5d = 16,000$

$a + 8d = 22,600$

$- \quad - \quad -$

$-3d = -6,600$

$d = 2,200$

Now, putting $d = 2,200$ in equation (i)

$a + 5d = 16,000$

$a + 5 \times 2,200 = 16,000$

$a + 11,000 = 16,000$

$a = 5,000$

(ii) Find the production during 8th year.

Sol. Production during 8th year is $(a + 7d)$

$= 5000 + 2(2200)$

$= 20400$

(iii) Find the production during first 3 years.

Sol. Production during first 3 year

$= 5000 + 7200 + 9400$

$= 21600$

(iv) In which year, the production is ₹ 29,200.

Sol. $N = 12$

Explanation: $a_n = 29,200$

$a + (n-1)d = 29,200$

$(x-1)2,900 = 29,200 - 5,000$

$2,200n - 2,200 = 24,200$

$2200n = 26,400$

$n = \frac{26,400}{2,200}$

$n = 12$

(v) Find the difference of the production during 7th year and 4th year.

Sol. Difference = 18200 – 11600 = 6600

Q. 2. **Your friend Veer wants to participate in a 200 m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds.**

[CBSE QB, 2021]

(i) Which of the following terms are in AP for the given situation

(a) 51, 53, 55.... **(b)** 51, 49, 47....

(c) – 51, – 53, – 55.... **(d)** 51, 55, 59...

Sol. Correct option: **(b).**

Explanation:

$$a = 51$$
$$d = -2$$
$$AP = 51, 49, 47 \ldots\ldots .$$

(ii) What is the minimum number of days he needs to practice till his goal is achieved ?

(a) 10 **(b)** 12

(c) 11 **(d)** 9

Sol. Correct option: **(c).**

Explanation: Goal = 31 second

$$n = \text{number of days}$$
$$\therefore \quad a_n = 31$$
$$a + (n-1)d = 31$$
$$51 + (n-1)(-2) = 31$$
$$51 - 2n + 2 = 31$$
$$-2n = 31 - 53$$
$$-2n = -22$$
$$n = 11$$

(iii) Which of the following term is not in the AP of the above given situation

(a) 41 **(b)** 30

(c) 37 **(d)** 39

Sol. Correct option: **(b).**

(iv) If n^{th} term of an AP is given by $a_n = 2n + 3$ then common difference of an AP is

(a) 2 **(b)** 3

(c) 5 **(d)** 1

Sol. Correct option: **(a).**

(v) The value of x, for which $2x, x + 10, 3x + 2$ are three consecutive terms of an AP

(a) 6 **(b)** – 6

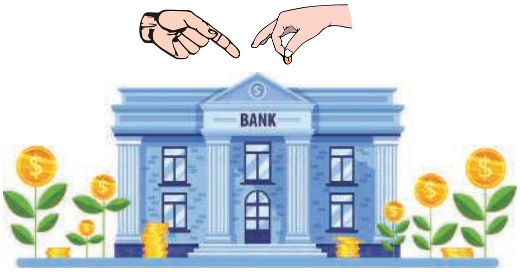
(c) 18 **(d)** – 18

Sol. Correct option: **(a).**

Explanation: Since, $2x, x + 10, 3x + 2$ are in AP, this common difference will remain same.

$$x + 10 - 2x = (3x + 2) - (x + 10)$$
$$10 - x = 2x - 8$$
$$2x = 18$$
$$x = 6$$

Q. 3. **Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of ₹ 1,18,000 by paying every month starting with the first instalment of ₹ 1000. If he increases the instalment by ₹ 100 every month, answer the following:** **[CBSE QB, 2021]**



(i) The amount paid by him in 30th installment is

(a) 3900 **(b)** 3500

(c) 3700 **(d)** 3600

Sol. Correct option: **(a).**

Explanation:

$$a = 1000$$
$$d = 100$$
$$a_{80} = a + (n-1)d$$
$$= 1000 + (30-1)100$$
$$= 1000 + 2900$$

(ii) The amount paid by him in the 30 installments is

(a) 37000 **(b)** 73500

(c) 75300 **(d)** 75000

Sol. Correct option: **(b).**

Explanation: Sum of 30 installments

$$= \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{30}{2}[2 \times 1000 + (30-1)100]$$
$$= 15[2000 + 2900]$$
$$= 15 \times 4900$$
$$= 73500$$

Total Amount paid in 30 installments = ₹ 73500

(iii) What amount does he still have to pay after 30th installment ?

(a) 45500 **(b)** 49000

(c) 44500 **(d)** 54000

Sol. Correct option: **(c).**

(iv) If total installments are 40 then amount paid in the last installment ?

(a) 4900 **(b)** 3900

(c) 5900 **(d)** 9400

Sol. Correct option: **(a).**

Explanation: Amount paid in 40th installment, a_{40}

$$= a + (n-1)d$$
$$= 1000 + (40-1)100$$
$$= 1000 + 3900$$
$$= 5900$$

(v) **The ratio of the 1st installment to the last installment is**

(a) **1 : 49** (b) **10 : 49**

(c) **10 : 39** (d) **39 : 10**

Sol. Correct option: **(b).**

AI **Q. 4. Jaspal Singh takes a loan from a bank for his car. Jaspal Singh repays his total loan of ₹ 118000 by paying every month starting with the first installment of ₹ 1000. If he increases the installment by ₹ 100 every month.**

(i) **If the given problem is based on A.P., then what is the first term and common difference ?**

(a) **1000, 100** (b) **100, 1000**

(c) **100, 100** (d) **1000, 1000**

Sol. Correct option: **(a).**

Explanation: The number involved in this case form an A.P. in which first term $(a) = 1000$ and common difference $(d) = 100$.

(ii) **The amount paid by him in 25th installment is:**

(a) **₹ 3300** (b) **₹ 3200**

(c) **₹ 3400** (d) **₹ 3500**

Sol. Correct option: **(c).**

Explanation: The amount paid by him in 25th installment is:

$$T_{25} = a + 24d$$
$$= 1000 + 24 \times 100$$
$$= 1000 + 2400$$
$$= ₹\ 3400.$$

(iii) **The amount paid by him in 30th installment is**

(a) **₹ 3900** (b) **₹ 3500**

(c) **₹ 3000** (d) **₹ 3600**

Sol. Correct option: **(a).**

Explanation: The amount paid by him in 30th installment,

$$T_{30} = a + 29d$$
$$= 1000 + 29 \times 100$$
$$= 1000 + 2900$$
$$= ₹\ 3900.$$

(iv) **The total amount paid by him in 25th and 30th installment is:**

(a) **₹ 7500** (b) **₹ 7300**

(c) **₹ 7800** (d) **₹ 7600**

Sol. Correct option: **(b).**

Explanation: Total amount paid by him in 25th and 30th installment = ₹ (3400 + 3900)

= ₹ 7300.

(v) **The difference amount paid by him in 26th and 28th installment is:**

(a) **₹ 400** (b) **₹ 100**

(c) **₹ 500** (d) **₹ 200**

Sol. Correct option: **(d).**

Explanation: The amount paid by him in 26th installment,

$$T_{26} = a + 25d$$
$$= 1000 + 25 \times 100$$
$$= 1000 + 2500$$
$$= ₹\ 3500$$

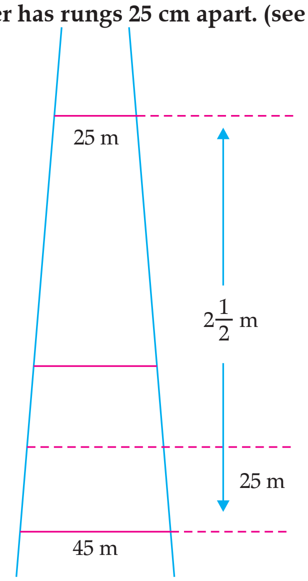
The amount paid by him in 28th installment,

$$T_{28} = a + 27d$$
$$= 1000 + 27 \times 100$$
$$= 1000 + 2700$$
$$= ₹\ 3700$$

∴ The difference amount paid by him in 26th and 28th installment is:

$$= ₹\ (3700 - 3500)$$
$$= ₹\ 200.$$

Q. 5. A ladder has rungs 25 cm apart. (see the below).



The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. The top and the bottom rungs are $2\frac{1}{2}$ m apart.

(i) **The top and bottom rungs are apart at a distance:**

(a) **200 cm** (b) **250 cm**

(c) **300 cm** (d) **150 cm**

Sol. Correct option: **(b).**

Explanation: Since the top and the bottom rungs are apart by $2\frac{1}{2}\text{ m} = \frac{5}{2}\text{ m}$

$$= \frac{5}{2} \times 100 \text{ cm}$$
$$= 250 \text{ cm}$$

(ii) **Total number of the rungs is:**

(a) **20** (b) **25**

(c) **11** (d) **15**

Sol. Correct option: **(c).**

Explanation: The distance between the two rungs is 25 cm.

Hence, the total number of rungs $= \dfrac{250}{25} + 1$

$= 11.$

(iii) The given problem is based on A.P. find its first term.

(a) 25 **(b) 45**

(c) 11 **(d) 13**

Sol. Correct option: **(a)**.

Explanation: The length of the rungs increases from 25 to 45 and total number of rungs is 11.

Thus, this is in the form of an A.P., whose first term is 25.

(iv) What is the last term of A.P. ?

(a) 25 **(b) 45**

(c) 11 **(d) 13**

Sol. Correct option: **(b)**.

Explanation: Total number of terms, $n = 11$ and the last term, $T_{11} = 45$.

(v) What is the length of the wood required for the rungs ?

(a) 385 **(b) 538**

(c) 532 **(d) 382**

Sol. Correct option: **(a)**.

Explanation: The required length of the wood,

$$S_{11} = \frac{11}{2}[25 + 45]$$

$$= \frac{11}{2} \times 70$$

$$= 385 \text{ cm.}$$

SELF ASSESSMENT TEST - 2

Maximum Time: 1 hour **MM: 25**

Q. 1. For what value of k, do the equation $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines ? R

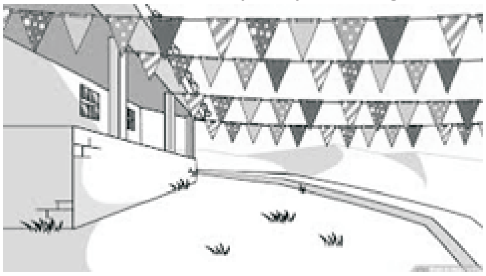
Q. 2. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, then find the product of the other two zeroes. R

Q. 3. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and – 3, then find the value of a. R

Q. 4. If in the equation $x + 2y = 10$, the value of y is 6, then find the value of x. R

Q. 5. Find the value of p for which $3x^2 - 5x + p = 0$ has equal roots. R

Q. 6. The students of a school decided to beautify the school on the Annual Day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time.



(i) What is the position of middle most flag ?

(a) 13th (b) 13.5th
(c) 14th (d) 12.5th

(ii) How many flags are left and right to the middle flag ?

(a) 14, 12 (b) 13, 13
(c) 13, 14 (d) 14, 13

(iii) How much distance did she cover in completing this job and returning back to collect her books ?

(a) 339 m (b) 634 m
(c) 364 m (d) 346 m

(iv) What is the maximum distance she travelled carrying a flag ?

(a) 13 m (b) 52 m
(c) 27 m (d) 26 m

(v) What is the mathematical concept related to this question ?

(a) A.P. (b) Lines
(c) Linear equations (d) none of these

Q. 7. If p and q are the zeroes of polynomial $f(x) = 2x^2 - 7x + 3$, find the value of $p^2 + q^2$.

AI Q. 8. Find the sum of the integers between 100 and 200 that are divisible by 6. A [Board Term-2, 2012]

Q. 9. How many three digit numbers are such that when divided by 7, leave a remainder 3 in each case ?
[Board Term-2, 2012 Set (1)]

Q. 10. If $(x^2 + y^2)(a^2 + b^2) = (ax + by)^2$. Prove that $\frac{x}{a} = \frac{y}{b}$.
A [Bord Term-2, 2014]

AI Q. 11. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and the breadth is increased by 3 units. The area is increased by 67 square units if length is increased by 3 units and breadth is increased by 2 units. Find the perimeter of the rectangle. A